

Name: _____

Directions: In order to receive full credit, you must show all relevant work. Solve all equations for all real (rational and/or irrational) solutions and/or imaginary solutions. Give all irrational solutions in simplified radical form. Give all imaginary solutions in i-form.

1. Factor $x^2 + 6x - 27$

$$(x+9)(x-3)$$

2. Solve $x^2 + 36 = 0$

$$x^2 = -36$$

$$\sqrt{x^2} = \pm \sqrt{-36}$$

$$x = \pm i\sqrt{36}$$

$$x = \pm 6i$$

$$\{6i, -6i\}$$

3. Solve $m^2 - 4m - 7 = 0$

$$m = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-7)}}{2} = \frac{4 \pm \sqrt{16+28}}{2} = \frac{4 \pm \sqrt{44}}{2} = \frac{4 \pm 2\sqrt{11}}{2}$$

$$\{2+\sqrt{11}, 2-\sqrt{11}\}$$

4. Solve $3y^3 - 24y = 0$

$$3y(y^2 - 8) = 0$$

$$3y = 0 \quad y^2 - 8 = 0$$

$$y = 0 \quad y^2 = 8$$

$$\sqrt{y^2} = \pm \sqrt{8}$$

$$y = \pm 2\sqrt{2}$$

$$\{0, 2\sqrt{2}, -2\sqrt{2}\}$$

5. Solve $3x^2 - 5x - 28 = 0$

$$(3x+7)(x-4) = 0$$

$$3x+7=0 \quad x-4=0$$

$$3x=-7 \quad x=4$$

$$\{-\frac{7}{3}, 4\}$$

6. Solve $x^4 + 2x^2 - 63 = 0$

$$(x^2+9)(x^2-7) = 0$$

$$x^2+9=0 \quad x^2-7=0$$

$$x^2=-9 \quad x^2=7$$

$$x = \pm \sqrt{-9} \quad x = \pm \sqrt{7}$$

$$x = \pm i\sqrt{9}$$

$$x = \pm 3i$$

$$\{3i, -3i, \sqrt{7}, -\sqrt{7}\}$$

7. Solve $x^3 - 7x^2 + 9x - 63 = 0$

$$x^2(x-7) + 9(x-7) = 0$$

$$(x-7)(x^2+9) = 0$$

$$x-7=0 \quad x^2+9=0$$

$$x^2=-9 \quad x = \pm \sqrt{-9}$$

$$\{7, 3i, -3i\}$$

8. Write an equation in standard form for the degree 2 polynomial function with the given

solution set $\{\sqrt{7}, -\sqrt{7}\}$

$$\begin{aligned} x &= \sqrt{7} & x &= -\sqrt{7} \\ x - \sqrt{7} &= 0 & x + \sqrt{7} &= 0 \\ (x - \sqrt{7})(x + \sqrt{7}) &= 0 \end{aligned}$$

$$\begin{aligned} x^2 + \cancel{\sqrt{7}x} - \cancel{\sqrt{7}x} - \sqrt{49} &= 0 \\ x^2 - 7 &= 0 \end{aligned}$$

9. A polynomial function of degree 3 has the solution set $\left\{-4, \frac{1}{3}\right\}$ with -4 as a double root. Determine an equation of the function in standard form.

$$\begin{aligned} x &= -4 & x &= -4 & x &= \frac{1}{3} \\ x + 4 &= 0 & x + 4 &= 0 & 3x - 1 &= 0 \\ (x + 4)(x + 4)(3x - 1) &= 0 \\ (3x - 1)(x^2 + 8x + 16) &= 0 \end{aligned}$$

$$\begin{aligned} 3x^3 + 24x^2 + 48x - x^2 - 8x - 16 &= 0 \\ 3x^3 + 23x^2 + 40x - 16 &= 0 \end{aligned}$$

10. Write an equation in standard form for the degree 3 polynomial function with the given solution set $\{1, 4i, -4i\}$

$$i^2 = -1$$

$$\begin{aligned} x &= 1 & x &= 4i & x &= -4i \\ x - 1 &= 0 & x - 4i &= 0 & x + 4i &= 0 \\ (x - 1)(x - 4i)(x + 4i) &= 0 \\ (x - 1)(x^2 + 4ix - 4ix - 16i^2) &= 0 \\ (x - 1)(x^2 + 16) &= 0 \\ x^3 - x^2 + 16x - 16 &= 0 \end{aligned}$$

11. Simplify $\frac{8y}{7} - \frac{y+28}{7} = \frac{8y - (y+28)}{7} = \frac{8y - y - 28}{7} = \frac{7y - 28}{7} = y - 4$

12. Simplify: $\frac{w+4}{9} + \frac{w-1}{18} = \frac{2(w+4)}{2 \cdot 9} + \frac{w-1}{18} = \frac{2w+8+w-1}{18} = \frac{3w+7}{18}$

13. Simplify: $\frac{m}{6} + \frac{n}{7} = \frac{7 \cdot m}{7 \cdot 6} + \frac{6 \cdot n}{6 \cdot 7} = \frac{7m+6n}{42}$

$$14. \text{ Simplify } \frac{5}{x} - \frac{3}{x+2} = \frac{5(x+2)}{x(x+2)} - \frac{3 \cdot x}{(x+2)x} = \frac{5x+10-3x}{x(x+2)} = \frac{2x+10}{x^2+2x}$$

$$15. \text{ Simplify: } \frac{6x^2-1}{6x^2} + \frac{(3-2x)x}{6x \cdot x} = \frac{6x^2-6}{6x^2} + \frac{3x-2x^2}{6x^2} = \frac{4x^2+3x-6}{6x^2}$$

$$16. \text{ Simplify } \frac{9y+15}{3} = 3y+5$$

$$17. \text{ Simplify } \frac{14k+7}{21} = \frac{2k+1}{3}$$

$$18. \text{ Simplify } \frac{y-8}{8-y} = -1$$

$$19. \text{ Simplify } \frac{m^2+9}{m+3} \text{ already simplified}$$

$$20. \text{ Simplify } \frac{3x+8}{9x^2-64} = \frac{3x+8}{(3x-8)(3x+8)} = \frac{1}{3x-8}$$

21. Simplify $\frac{3m^2 + 14m - 5}{3m^2 - 10m + 3} = \frac{(3m - 1)(m + 5)}{(3m - 1)(m - 3)}$

22. Simplify $\frac{k^3 + 64}{k + 4} = \frac{(k + 4)(k^2 - 4k + 16)}{k + 4} = k^2 - 4k + 16$

23. Simplify $\frac{y^2 + 3y - 18}{y^2 - 10y + 21} \times \frac{y^2 - 4y - 21}{y^2 + 8y + 12} = \frac{(y + 6)(y - 3)(y - 7)(y + 3)}{(y - 3)(y - 7)(y + 2)(y + 6)} = \frac{y + 3}{y + 2}$

24. Simplify $\frac{x^2 - 25}{x^2 - 10x} \div \frac{x^2 + 10x + 25}{x^2 + 5x} = \frac{(x - 5)(x + 5)}{x(x - 10)} \cdot \frac{x(x + 5)}{(x + 5)(x + 5)} = \frac{x - 5}{x - 10}$

25. State all real number restrictions on $x: \frac{x^2 - 49}{x^2 + 100}$
 $x^2 + 100$ cannot equal zero by substituting real numbers
 there are no restrictions

26. State all real number restrictions on $x: \frac{x^2 - 3x - 18}{5x^3 - 20x}$
 $5x^3 - 20x = 5x(x^2 - 4) = 5x(x - 2)(x + 2)$

$$5x \neq 0$$

$$x \neq 0$$

$$x - 2 \neq 0$$

$$x \neq 2$$

$$x + 2 \neq 0$$

$$x \neq -2$$

27. Solve $\frac{4}{6} = \frac{2w}{21}$

$$\frac{2}{3} = \frac{2w}{21}$$

$$2(21) = 3(2w)$$

$$42 = 6w$$

$$7 = w$$

$$\{7\}$$

28. Solve $\frac{6}{x+2} = \frac{9}{x+3}$

$$(x+3)(6) = (x+2)(9)$$

$$6x + 18 = 9x + 18$$

$$6x = 9x$$

$$0 = 3x$$

$$0 = x$$

$$\{0\}$$

29. Solve $\frac{9}{10} - y = \frac{4y}{5}$

$$10 \cdot \frac{9}{10} - 10y = 10 \cdot \frac{4y}{5}$$

$$9 - 10y = 2 \cdot 4y$$

$$9 = 18y$$

$$\frac{1}{2} = y$$

30. Solve $\frac{21}{h^2} - 1 = \frac{4}{h}$

$$h^2 \cdot \frac{21}{h^2} - h^2 \cdot 1 = h^2 \cdot \frac{4}{h}$$

$$21 - h^2 = 4h$$

$$0 = h^2 + 4h - 21$$

$$0 = (h+7)(h-3)$$

$$h+7=0 \quad h-3=0$$

$$h=-7 \quad h=3$$

$$\{-7, 3\}$$

31. Solve $1 + \frac{2}{y} = \frac{4}{y-3}$

$$y(y-3) + y(y-3) \frac{2}{y} = y(y-3) \frac{4}{y-3}$$

$$y^2 - 3y + 2(y-3) = 4y$$

$$y^2 - 3y + 2y - 6 = 4y$$

$$y^2 - 5y - 6 = 0$$

$$(y-6)(y+1) = 0$$

$$y-6=0 \quad y+1=0$$

$$y=6 \quad y=-1$$

$$\{6, -1\}$$

Optional Extra Credit: Solve $\frac{2}{x+1} - \frac{1}{x} = \frac{1}{6}$

$$6x(x+1) \cdot \frac{2}{x+1} - 6x(x+1) \cdot \frac{1}{x} = 6x(x+1) \cdot \frac{1}{6}$$

$$6x \cdot 2 - 6(x+1) = x(x+1)$$

$$12x - 6x - 6 = x^2 + x$$

$$0 = x^2 - 5x + 6$$

$$0 = (x-2)(x-3)$$

$$\{2, 3\}$$

